

Course of Study Template (version B - template)

Overview:

Title of Course: Trigonometry/Statistics	
Course Author(s): David Wetzel	Schools where the course will be taught: Tamalpais High School
Length of Course: Year	Subject Area and Discipline: Math
Grade Levels: 11/12	Is this course an integrated course? No
Is this course being submitted for possible UC honors designation? No	Are you seeking UC approval? If so, in what area (A-G)? Yes, 'C'
Prerequisites (required or recommended): Advanced Algebra or Intermediate Algebra	Co-requisites (required or recommended): Geometry
<p>Check all that apply:</p> <p><input checked="" type="checkbox"/> UC A-G course</p> <p><input type="checkbox"/> Graduation Requirement</p> <p><input checked="" type="checkbox"/> Elective</p> <p><input type="checkbox"/> Honors/AP</p> <p><input type="checkbox"/> ROP</p>	

Introduction

Course Overview: Provide a brief summary/snapshot of the courses' content. Include overarching goals for the course (3-5 sentences).

Trigonometry is a semester course that covers all the State Standards.

Statistics is a semester course that covers all the State Standards. Together they form a one-year course for students that have already completed their second year of Algebra and is an alternative to Pre-Calculus. Over the course of the year, students will participate in a broad range of instructional experiences using a variety of methods including the Common Core mathematical practices. The essential program goals for Trigonometry are to interpret and graph trigonometric functions, generate and use the Unit Circle, verify and use trigonometric identities, complete triangles using the law of sines and cosines and solve real life scenarios using trigonometry. The essential program goals for Statistics are to analyze and present data using charts and graphs and spreadsheets, measure center and variability, create and analyze boxplots, determine and create trend lines and correlations, find probabilities in a standard normal distribution, calculate confidence intervals, calculate

z-scores and percentiles, perform significance tests and perform a chi square test. Successful completion of this course will prepare students for Pre-Calculus or AP Statistics. The intended audience is all students who are intending to continue their education at a four year college or university.

Stage 1 Desired Results

Unit Title: Trigonometric Functions

ESTABLISHED LEARNING GOALS

National Common
Core Standards:

*F-IF 7e

*G-SRT 6-11

*F-TF 2.1, 5

Transfer

Students will be able to independently use their learning to...

- Determine the amplitude, direction, period and position of trigonometric functions
- Compare the graphs of parent and transformed trigonometric functions
- Accurately graph a trigonometric function (parent and transformed)
- Write the equation of the trigonometric function given key features of a graph,
- Use a trigonometric function in a real-world application

Meaning Making

UNDERSTANDINGS

Students will understand that...
Trigonometric Functions can not only be used to solve triangles but can be used to model real life scenarios.

ESSENTIAL QUESTIONS

- *How do you determine which trigonometric function should be used to solve a triangle?
- *How can a trigonometric function model a real life scenario?

Acquisition

Students will know...

- *What trig ratio to use for a given triangle
- *What features of a trig function are implied in a real life scenario

Students will be skilled at...

- *Calculating sine, cosine and tangent
- *Graphing trig functions

5-10 sentence summary of key learning in this unit.

As students learn the characteristics of trig ratios, they will practice applying their knowledge to different triangles. They are expected to select the appropriate trig ratio and then calculate it accurately.

Students will be introduced to the characteristics of the parent functions of all 6 trig functions. They will practice applying what they have learned to functions with transformations. The more each student demonstrates understanding of each transformation, more variations will be added to each problem. Students will then demonstrate their understanding by writing trig functions directly from characteristics.

Stage 2 - Evidence

Learning Goals

Measured:

**can be referenced by number*

Success Criteria (e.g., Learning progression, rubric, proficiency scale, etc.)

Skill:	Level 3:	Level 2:	Level 1:
Trig Ratios	Draw a picture to represent scenario. Accurately select appropriate trig ratio. Accurately solve for missing angles and sides of a triangle.	Select appropriate trig ratio but minor calculation errors. Calculations are accurate but scenario was drawn incorrectly.	Appropriate trig ratio not selected and or solved incorrectly.
Trig Functions	Graph and label function accurately. Identify all changes from parent function. Accurately model a scenario with an appropriate trig function.	Sketching trig function accurately without clarifying characteristics. Able to transform a parent function without identifying the transformations. (table only)	Accurate graph of wrong trig function. (graphing sine when it should be cosine etc.)

Sample Assessment (e.g. Performance tasks, anchor of student work, common assessment etc.)

Quiz—Trig. Function Transformations

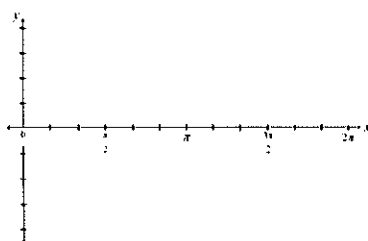
Name _____ Per _____

Trig/Stat—Fall

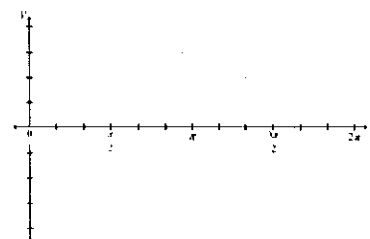
v.3

~~~Graph at least one period of each function and label the 5 critical values.~~~

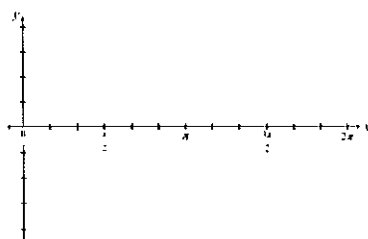
1)  $y = -\tan \frac{\theta}{2}$



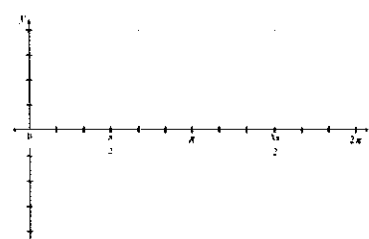
2)  $y = \csc \left( \theta - \frac{\pi}{4} \right) + 2$



3)  $y = 3\sin \left( \theta - \frac{\pi}{4} \right) - 1$



4)  $y = \cos 2 \left( \theta - \frac{\pi}{4} \right)$



## Stage 3 – Learning Plan

Learning Goals

Addressed:

\*F-TF 2.1, 5

### Sample Assignment:

Worksheet based assignment:

- \* Students are asked to graph all 6 parent trig functions.
- \* Students are then asked to graph a set of functions that have 1 transformation.
- \* Students are then asked to graph a set of functions with 2 transformations.
- \* Students are then asked to graph a set of functions with 3 or more transformations.
- \* Finally, students are asked to create a graph that fits either a scenario or a description of transformations.

(Students progress through these worksheets at their own pace or in small groups)

(How fast, how accurate and how far students progress through the worksheets should be used to inform future lessons and interventions)

|  |                                                                                                                                                                                                                                                                                                                                  |
|--|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|  | <p><b>Differentiated Approaches:</b></p> <p>Small groups work well for this lesson as students will have different levels of difficulty but will all be the same transformations. Students can work together to help each other improve their skill level whether they are working on 1 transformation of 5 transformations.</p> |
|--|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

| Stage 1 Desired Results                                                          |                                                                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                                                                  |
|----------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Unit Title: Unit Circle                                                          |                                                                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                                                                  |
| ESTABLISHED<br>LEARNING GOALS<br>National Common<br>Core Standards:<br>*F-TF 1,2 | Transfer                                                                                                                                                                                                                                                                                                                                                                             |                                                                                                                                                                                                  |
|                                                                                  | Students will be able to independently use their learning to... <ul style="list-style-type: none"><li>• Generate a Unit circle labeling degrees, radians and ordered pairs</li><li>• Use the unit circle to evaluate all 6 trigonometric functions</li><li>• Convert radians to degrees and degrees to radians</li><li>• Calculate positive and negative coterminal angles</li></ul> |                                                                                                                                                                                                  |
|                                                                                  | Meaning Making                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                                                                                                                  |
|                                                                                  | UNDERSTANDINGS<br>Students will understand that... <ul style="list-style-type: none"><li>*Trig ratios are directly related to the unit circle.</li><li>*Terminal sides of an angle can be defined using angles, ordered pairs or radians when using the unit circle.</li></ul>                                                                                                       | ESSENTIAL QUESTIONS <ul style="list-style-type: none"><li>*How can you identify a given position on the unit circle?</li><li>*What impact does direction have on defining rotation?</li></ul>    |
|                                                                                  | Acquisition                                                                                                                                                                                                                                                                                                                                                                          |                                                                                                                                                                                                  |
|                                                                                  | Students will know ... <ul style="list-style-type: none"><li>*How to use the Unit Circle to find common trig ratios.</li><li>*How to use the Unit Circle to calculate positive and negative coterminal angles.</li></ul>                                                                                                                                                             | Students will be skilled at... <ul style="list-style-type: none"><li>*Evaluating trig ratios</li><li>*Converting degrees to radians and vice versa</li><li>*Recreating the Unit Circle</li></ul> |

5-10 sentence summary of key learning in this unit.

Students will be introduced to the unit circle by showing its relationship to special right triangles. They will then be expected to reproduce the unit circle from memory. Each student will be able to use the unit circle to identify sine and cosine values for common point and then use those values to calculate the 4 remaining trig ratios. Students will be able to identify terminal sides rotating both clockwise and counterclockwise.

## Stage 2 - Evidence

Learning Goals  
Measured:  
\*F-TF 1,2

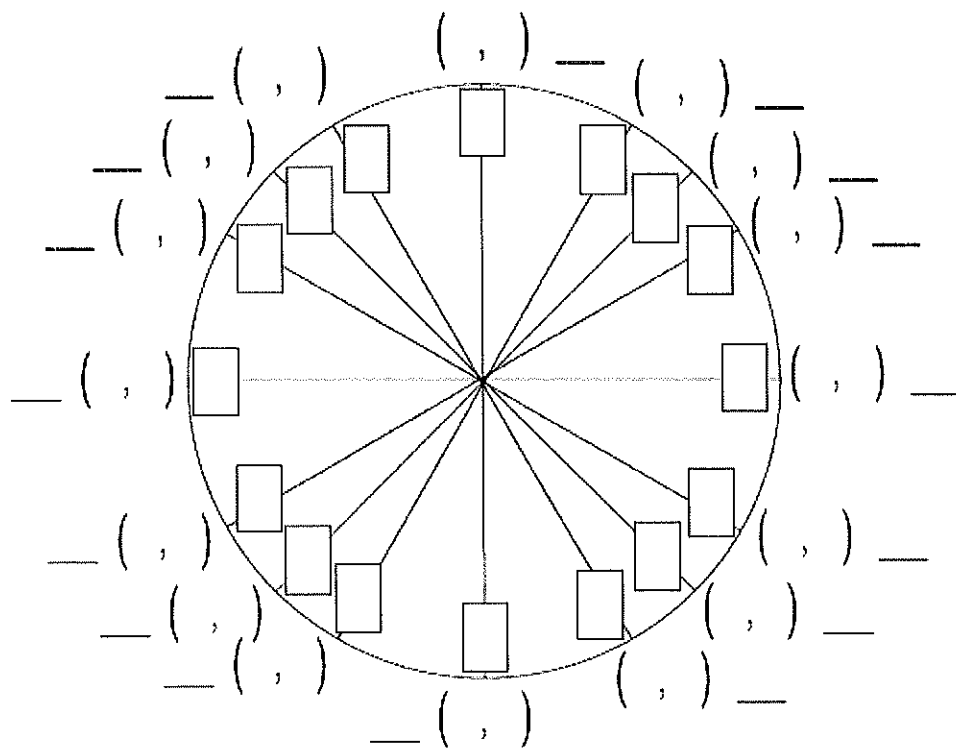
**Success Criteria** (e.g., Learning progression, rubric, proficiency scale, etc.)

| Skill:                   | Level 3:                                                                                                                         | Level 2:                                                                                                                    | Level 1:                                                                                    |
|--------------------------|----------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------|
| Generate the Unit Circle | Circle is drawn to scale with all primary values are accurately marked in radians, degrees and ordered pairs.                    | Circle is drawn but values are not to scale. Radians, degrees and ordered pairs are identified but not completely accurate. | Values are missing and or not accurate.                                                     |
| Trig Ratios              | All 6 trig ratios are calculated accurately from the unit circle and all calculations are shown as variation of sine and cosine. | Sine and cosine are identified accurately but some errors in the remaining trig ratios.                                     | Sine and cosine not accurately identified thus making the remaining trig ratios inaccurate. |
| Conversions              | Accurate conversion of radians to degrees and degrees to radians with all final results in reduced form.                         | Accurate conversion of radians to degrees and degrees to radians with all final results not in reduced form.                | Inaccurate conversions or reducing results done improperly.                                 |

**Sample Assessment** (e.g. Performance tasks, anchor of student work, common assessment etc.)

*Unit Circle Test*    Part 1    Name \_\_\_\_\_ per \_\_\_\_\_

Fill in the Unit Circle. Put: radians in the boxes,  
(ordered pairs) in the parentheses & degrees on the lines. (10pts)

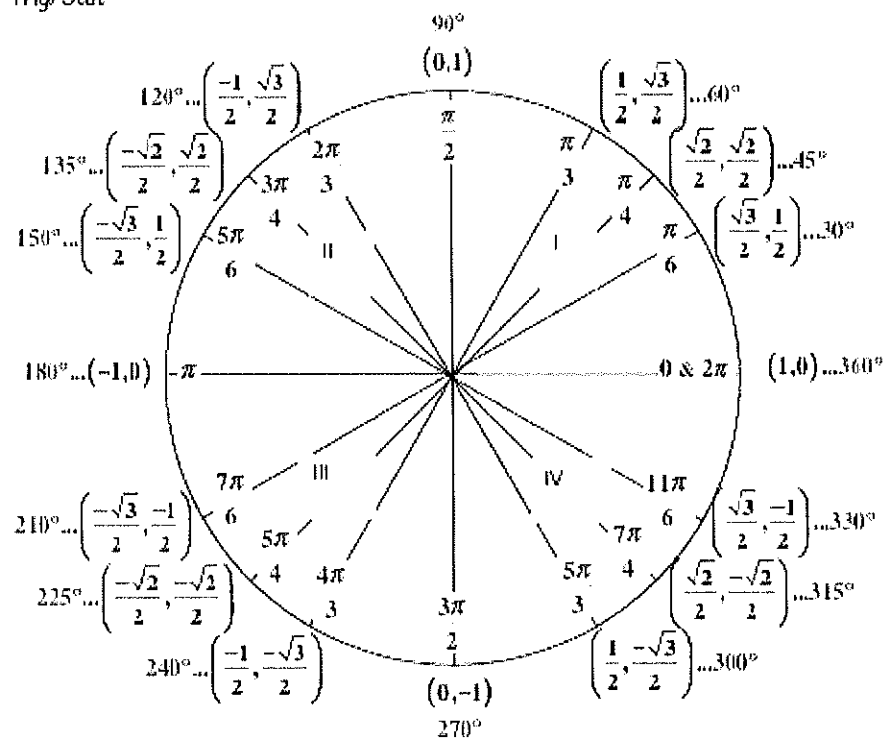


# Test—Unit Circle

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Name \_\_\_\_\_ per \_\_\_\_\_

Trig/Stat



(1-12)

Use the Unit Circle to fill in the chart below:

| $\theta$      | $\pi$ | $\frac{7\pi}{6}$ | $\frac{5\pi}{4}$ | $\frac{4\pi}{3}$ | $\frac{3\pi}{2}$ |
|---------------|-------|------------------|------------------|------------------|------------------|
| $\sin \theta$ |       |                  |                  |                  |                  |
| $\cos \theta$ |       |                  |                  |                  |                  |
| $\tan \theta$ |       |                  |                  |                  |                  |



Use the Unit Circle to find each value:

v.3

|                             |                             |                                         |
|-----------------------------|-----------------------------|-----------------------------------------|
| 13) $\sin(315^\circ) =$     | 14) $\tan 240^\circ =$      | 15) $\cos 150^\circ =$                  |
| 16) $\csc \frac{5\pi}{4} =$ | 17) $\tan \frac{5\pi}{3} =$ | 18) $\cos\left(-\frac{\pi}{6}\right) =$ |

Convert the following values from Degrees to Radians: (leave in simplest form)

|                 |                 |                 |
|-----------------|-----------------|-----------------|
| 19) $180^\circ$ | 20) $155^\circ$ | 21) $450^\circ$ |
|-----------------|-----------------|-----------------|

Convert the following values from Radians to Degrees: (leave in simplest form)

|                      |         |             |
|----------------------|---------|-------------|
| 22) $\frac{7\pi}{6}$ | 23) $9$ | 24) $12\pi$ |
|----------------------|---------|-------------|

Indicate what quadrant each value falls in:

|                  |                  |                    |
|------------------|------------------|--------------------|
| 25) $1200^\circ$ | 26) $12$ radians | 27) $-297$ degrees |
|------------------|------------------|--------------------|

### Stage 3 – Learning Plan

Learning Goals  
Addressed:  
\*F-TF 1,2

#### Sample Assignment:

Students are given a template of a 30-60-90 triangle and a 45-45-90 triangle, both with a hypotenuse of 1 unit. They work together to label the remaining sides of each triangle on the front and the back of each triangle. Following that, they use the triangles along with a template of a circle that has a radius of 1 unit. Students use both triangles to work their way around the circle marking 30, 45 and 90 degrees in each quadrant. Upon completion of this exercise, students will have a unit circle with ordered pairs and degrees marked. In discussion, they also determine the radian values for each point.

|  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
|--|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|  | <p><b>Differentiated Approaches:</b></p> <p>This assignments leads itself to multiple levels of discovery and intervention if needed. As students are working on the triangles, review sheets from geometry can be used to help them remember the special right triangle values. As students are marking the points on the unit circle review of the coordinate plane can be helpful. Providing students with a protractor can also be helpful for identifying each angle marked on the circle. By working together, students can get support at any phase of the exercise. If further interventions are needed, teacher can work with students individually.</p> |
|--|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

| Stage 1 Desired Results                                                                                      |                                                                                                                                                                                                                                                                                       |                                                                                                                                                                                           |
|--------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Unit Title: Trigonometric Identities</b>                                                                  |                                                                                                                                                                                                                                                                                       |                                                                                                                                                                                           |
| <p>ESTABLISHED LEARNING GOALS<br/>(e.g. standards at the local, state and/or national level)<br/>*F-TF 8</p> | <b>Transfer</b>                                                                                                                                                                                                                                                                       |                                                                                                                                                                                           |
|                                                                                                              | <p><i>Students will be able to independently use their learning to...</i></p> <ul style="list-style-type: none"> <li>Verify trigonometric identities</li> <li>Provide reasoning for the proof of a trigonometric identities</li> <li>Evaluate expressions using identities</li> </ul> |                                                                                                                                                                                           |
|                                                                                                              | <b>Meaning Making</b>                                                                                                                                                                                                                                                                 |                                                                                                                                                                                           |
|                                                                                                              | <p><b>UNDERSTANDINGS</b><br/><i>Students will understand that...</i><br/>*Trigonometric identities can be change in form but remain equal.<br/>*Expressions can be altered to look like trigonometric identities.</p>                                                                 | <p><b>ESSENTIAL QUESTIONS</b><br/>*What alterations can be made to a trigonometric identity?<br/>*What operations can be performed on an expression to make it look like an identity?</p> |
|                                                                                                              | <b>Acquisition</b>                                                                                                                                                                                                                                                                    |                                                                                                                                                                                           |
|                                                                                                              | <p><i>Students will know...</i><br/>*How to use algebraic and trigonometric properties to alter the appearance of an expression or identity.</p>                                                                                                                                      | <p><i>Students will be skilled at...</i><br/>*Referencing properties as they are used to alter expressions or identities.<br/>*Altering expressions and identities.</p>                   |

5-10 sentence summary of key learning in this unit.

This unit is an exercise of memory and application. As students work with identities and equations, they must remember, identify and use algebraic and trigonometric identities they have learned in previous courses. Problems will be introduced that only require a few steps/properties to attain the desired format. As students master the problems, they will be introduced to problems that require more steps and more properties. Once they are skilled at these manipulations, they will be asked to apply their knowledge to actual problems that arrive at numeric answers.

## Stage 2 - Evidence

Learning Goals  
Measured:  
\*F-TF 8

**Success Criteria** (e.g., Learning progression, rubric, proficiency scale, etc.)

| Skill:               | Level 3:                                                                                                                                       | Level 2:                                                                                                                                | Level 1:                                                             |
|----------------------|------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------|
| Verifying Identities | Students manipulate one side of an identity until it matches the other side. Each step uses an algebraic or trigonometric identity accurately. | Students manipulate both sides of an identity until both sides match. Each step uses an algebraic or trigonometric identity accurately. | Student manipulates identity but some steps are not accurate.        |
| Proofs               | Students manipulate one side of an identity until it matches the other side. An accurate property is identified and stated for each step.      | Students manipulate both sides of an identity until both sides match. An accurate property is identified and stated for each step.      | Properties and or steps are not accurately identified and or stated. |

**Sample Assessment** (e.g. Performance tasks, anchor of student work, common assessment etc.)

Test 4 Review - Trig Identities

Name \_\_\_\_\_

**MULTIPLE CHOICE.** Choose the one alternative that best completes the statement or answers the question.

Use the fundamental identities to simplify the expression.

1)  $\frac{\tan \theta}{\cot \theta}$

A)  $\cos^3 \theta$

B)  $\tan^2 \theta$

C)  $\sec^2 \theta$

D)  $\sin \theta$

2)  $\frac{\cos^2 \theta}{\sin^2 \theta} + \csc \theta \sin \theta$

A)  $\sec^2 \theta$

B)  $\csc^2 \theta$

C)  $\tan^2 \theta$

D) 1

3)  $\csc \theta (\sin \theta + \cos \theta)$

A)  $1 + \cot \theta$

B)  $\sec \theta \csc \theta$

C)  $-2 \tan^2 \theta$

D)  $\sin \theta \tan \theta$

4)  $\frac{(\sin \theta + \cos \theta)^2}{1 + 2 \sin \theta \cos \theta}$

A)  $-\sec^2 \theta$

B) 0

C) 1

D)  $1 - \sin \theta$

5)  $\frac{\csc \theta \cot \theta}{\sec \theta}$

A)  $\cot^2 \theta$

B)  $\sec^2 \theta$

C)  $\csc^2 \theta$

D) 1

6)  $\sin^2 \theta + \tan^2 \theta + \cos^2 \theta$

A)  $\sin \theta$

B)  $\sec^2 \theta$

C)  $\cos^3 \theta$

D)  $\tan^2 \theta$

**SHORT ANSWER.** Write the word or phrase that best completes each statement or answers the question.

Find the exact value by using a sum or difference identity.

7)  $\cos (165^\circ)$

8)  $\sin 15^\circ$

**MULTIPLE CHOICE.** Choose the one alternative that best completes the statement or answers the question.

Use trigonometric identities to find the exact value.

9)  $\sin 10^\circ \cos 50^\circ + \cos 10^\circ \sin 50^\circ$

A)  $\frac{\sqrt{3}}{3}$

B)  $\frac{1}{6}$

C)  $\frac{\sqrt{3}}{2}$

D)  $\frac{1}{2}$

10)  $\cos (310^\circ) \cos (50^\circ) - \sin (310^\circ) \sin (50^\circ)$

A) 0

B)  $\frac{\sqrt{2}}{2}$

C) -1

D) 1

$$11) \frac{\tan 155^\circ - \tan 35^\circ}{1 + \tan 155^\circ \tan 35^\circ}$$

A)  $-\sqrt{3}$

B)  $-2$

C)  $-\frac{\sqrt{3}}{3}$

D)  $-\frac{1}{2}$

Write in terms of the cofunction of a complementary angle.

12)  $\sin 85^\circ$

A)  $\csc 5^\circ$

B)  $\csc 95^\circ$

C)  $\cos 95^\circ$

D)  $\cos 5^\circ$

13)  $\cos \frac{\pi}{16}$

A)  $\sin \frac{7\pi}{16}$

B)  $\sin \frac{15\pi}{16}$

C)  $\sec \frac{15\pi}{16}$

D)  $\sec \frac{7\pi}{16}$

**SHORT ANSWER.** Write the word or phrase that best completes each statement or answers the question.

Find the exact value of the expression using the provided information.

14) Find  $\cos(B + C)$  given that  $\sin B = -\frac{1}{2}$ , with B in quadrant IV, and  $\sin C = \frac{1}{4}$ , with C in quadrant II.

15) Find  $\sin(A - B)$  given that  $\cos A = \frac{1}{3}$ , with A in quadrant I, and  $\sin B = -\frac{1}{2}$ , with B in quadrant IV.

**MULTIPLE CHOICE.** Choose the one alternative that best completes the statement or answers the question.

Find the exact functional value.

16) If  $\cos \alpha = -\frac{5}{13}$  and  $\tan \alpha < 0$ , then find  $\sin 2\alpha$ .

A)  $\frac{112}{169}$

B)  $-\frac{119}{169}$

C)  $\frac{120}{169}$

D)  $-\frac{120}{169}$

17) If  $\sin \theta = -\frac{4}{5}$  and  $\cot \theta < 0$ , then find  $\cos 2\theta$ .

A)  $\frac{24}{25}$

B)  $\frac{7}{25}$

C)  $-\frac{24}{25}$

D)  $-\frac{7}{25}$

Use an identity to write the expression as a single trigonometric function or as a single number.

18)  $\sin 22.5^\circ \cos 22.5^\circ$

A)  $\frac{\sqrt{2}}{4}$

B)  $\frac{\sqrt{2}}{2}$

C)  $\frac{\sqrt{3}}{3}$

D)  $\sqrt{3}$

19)  $2 \cos^2 75^\circ - 1$

A)  $\frac{1}{2}$

B)  $-\frac{\sqrt{3}}{2}$

C)  $\frac{\sqrt{3}}{2}$

D)  $-\frac{1}{2}$

### Stage 3 – Learning Plan

Learning Goals  
Addressed:  
\*F-TF 8

#### Sample Assignment:

Students are given a list of trigonometric identities. The identities progress in difficulty from 1-2 steps to 10 or more steps. They are asked to verify half of them by altering one side of the identity until it matches the other side. On the other half, they are asked to write down the corresponding algebraic or trigonometric property for each of their steps (proof). (Students can work alone or in small groups)

|  |                                                                                                                                                                                                                                                                                                                                                                                                     |
|--|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|  | <p><b>Differentiated Approaches:</b></p> <p>The variety of identities that are presented are done so at varying levels of difficulty. As students master 1-2 step identities, they can move on to identities that require 3-4 steps. Having students work in small groups will allow students to work together on identifying possible steps no matter how many are required for their problem.</p> |
|--|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

| Stage 1 Desired Results                                                                                                                                                                                                                                                                                                                                                                                     |                                                                                                                                                                                                                                                                                                                      |                                                                                                                      |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------|
| Unit Title: Data Graphs and Charts                                                                                                                                                                                                                                                                                                                                                                          |                                                                                                                                                                                                                                                                                                                      |                                                                                                                      |
| ESTABLISHED<br>LEARNING GOALS<br>(e.g. standards at the<br>local, state and/or<br>national level)<br>*6.SP 4-5<br><u>HSS.ID.A.1</u>                                                                                                                                                                                                                                                                         | Transfer                                                                                                                                                                                                                                                                                                             |                                                                                                                      |
|                                                                                                                                                                                                                                                                                                                                                                                                             | Students will be able to independently use their learning to... <ul style="list-style-type: none"><li>Differentiate categorical data from quantitative data</li><li>Create and analyze a frequency distribution table</li><li>Create and analyze a pie chart, bar-chart, dot plot, stem plot and histogram</li></ul> |                                                                                                                      |
|                                                                                                                                                                                                                                                                                                                                                                                                             | Meaning Making                                                                                                                                                                                                                                                                                                       |                                                                                                                      |
|                                                                                                                                                                                                                                                                                                                                                                                                             | UNDERSTANDINGS<br>Students will understand that...<br>*Different data can be represented in different ways.                                                                                                                                                                                                          | ESSENTIAL QUESTIONS<br>*What graph, table or chart is best for representing a given data?                            |
|                                                                                                                                                                                                                                                                                                                                                                                                             | Acquisition                                                                                                                                                                                                                                                                                                          |                                                                                                                      |
|                                                                                                                                                                                                                                                                                                                                                                                                             | Students will know...<br>*How to properly display data.                                                                                                                                                                                                                                                              | Students will be skilled at...<br>*Reading tables, charts and histograms.<br>*Drawing tables, charts and histograms. |
| 5-10 sentence summary of key learning in this unit.<br>Students will learn the different types of data and what display is best for each type.<br>Students will learn how to glean information from varying types of data displays like histograms, boxplots, pie charts and dot plots. For each display of data, students will learn how to read each, answer questions and predict future values of each. |                                                                                                                                                                                                                                                                                                                      |                                                                                                                      |

## Stage 2 - Evidence

Learning Goals

Measured:

\*6.SP 4-5

HSS.ID.A.1

**Success Criteria** (e.g., Learning progression, rubric, proficiency scale, etc.)

| Skill:                                          | Level 3:                                                                        | Level 2:                                                                                     | Level 1:                                        |
|-------------------------------------------------|---------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------|-------------------------------------------------|
| Crating a graph, chart, table, plot, histogram. | Display type is accurately selected, accurately created and labeled accurately. | Display is created and labeled accurately but not necessarily the best display for the data. | Display is not complete for the data presented. |

**Sample Assessment** (e.g. Performance tasks, anchor of student work, common assessment etc.)

Data Test

Name \_\_\_\_\_

Per. \_\_\_\_\_

1. The times (in minutes) of a runner's last 10 runs are 88, 65, 88, 49, 60, 71, 56, 87, 64 and 52.

a. Find the mean, median and mode for the times.

b. Create a box plot for the data set.

2. Use the two-way table to answer the following questions. Use each value, rounded to the nearest percent, only once.

|                                |            | Age   |       |
|--------------------------------|------------|-------|-------|
|                                |            | 15-21 | 35-41 |
| How You Look Up Spelling Words | Online     | 136   | 50    |
|                                | Dictionary | 20    | 66    |

a. The percent of 35-41 year olds in the survey that look up spelling words online.

b. The percent of 15-21 year olds in the survey.

c. Given that a person looks up words in a dictionary, the conditional relative frequency that he or she is 35-41 years old.

d. The percent of people in the survey who looks up words in a dictionary.

e. Given that a person is 35-41 years old, the conditional relative frequency that he or she looks up spelling words online.



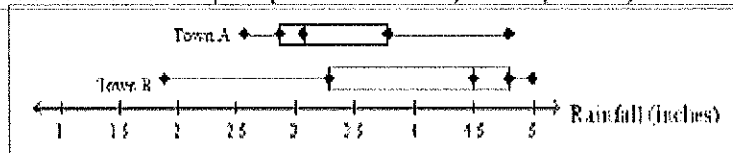
3. The scores for a golf tournament are shown in the table.

| Golf Scores |    |    |
|-------------|----|----|
| -1          | 4  | -2 |
| -2          | 0  | 1  |
| 4           | -2 | -2 |
| -2          | -2 | 4  |

- a. Find the mean, median, and mode of the golf scores.

- b. Which measure of center best represents the data? Explain.

4. The double box-and-whisker plot represents the monthly rainfalls (in inches) of two towns.



- a. Identify the shape of each distribution.

- b. Which measure of center is better to use for each town?

- c. Describe the spread (range and IQR) of each box plot.

- d. One other comment regarding comparing quartiles (25%, 50%, 75% or 100%) of the two boxplots.

### Stage 3 – Learning Plan

Learning Goals  
Addressed:  
\*6.SP 4-5  
HSS.ID.A.1

#### Sample Assignment:

Students are asked to select a topic of interest and survey their classmates for the answers to 1-2 questions. Students then decide what data display will best represent the data they collected. In small groups, students share their charts with each other and defend their choice of data display. All students are then expected to ask questions that were not directly shared. The student that created each display will use the data display to answer each question. (These displays can be created on poster paper or on a digital format that can be presented).

|  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
|--|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
|  | <p><b>Differentiated Approaches:</b> Include descriptions of how to meet the needs of diverse learners in the context of the sample assignment above (2-3 examples recommended).</p> <p>This unit is visual by nature so it leads to levels of differentiation. Students can be presented with graphs, charts and tables and be asked to answer questions from the already generated images. Students can also be given the information and be asked to generate the graphs, charts and tables. For the highest level of understanding, students can be asked to generate the images and also be required to answer questions that verify their responses.</p> |

| Stage 1 Desired Results                            |                                                                                                                                                                                                                                                                                          |                                                                                                                                                                                                                   |
|----------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Unit Title: Center and Variability                 |                                                                                                                                                                                                                                                                                          |                                                                                                                                                                                                                   |
| ESTABLISHED<br>LEARNING GOALS<br><u>HSS.ID.A.4</u> | <i>Transfer</i>                                                                                                                                                                                                                                                                          |                                                                                                                                                                                                                   |
|                                                    | <i>Students will be able to independently use their learning to...</i> <ul style="list-style-type: none"><li>• Calculate Variance and Standard Deviation</li><li>• Calculate median, quartiles, interquartile range, max, min and outliers</li><li>• Make and compare boxplots</li></ul> |                                                                                                                                                                                                                   |
|                                                    | <i>Meaning Making</i>                                                                                                                                                                                                                                                                    |                                                                                                                                                                                                                   |
|                                                    | UNDERSTANDINGS<br><i>Students will understand that...</i> <ul style="list-style-type: none"><li>*Data can be represented in a variety of ways.</li></ul>                                                                                                                                 | ESSENTIAL QUESTIONS <ul style="list-style-type: none"><li>*What image is best for medians?</li><li>*What information can be gleaned from a boxplot?</li><li>*How is standard deviation calculated?</li></ul>      |
|                                                    | <i>Acquisition</i>                                                                                                                                                                                                                                                                       |                                                                                                                                                                                                                   |
|                                                    | <i>Students will know...</i> <ul style="list-style-type: none"><li>*Answer questions from a boxplot.</li><li>*Calculate all critical characteristics of a boxplot.</li></ul>                                                                                                             | <i>Students will be skilled at...</i> <ul style="list-style-type: none"><li>*Calculating variance and standard deviation.</li><li>*Calculating medians, interquartile range and upper and lower bounds.</li></ul> |

5-10 sentence summary of key learning in this unit.

Analyzing data through the use of boxplots requires quite a few different calculations. Students will first be taught the characteristics of a boxplot, how to identify values simply by looking at the boxplot and how to answer questions from the image. Following that, students will be taught how to make all the calculations that are represented on a boxplot. The better students get at identifying and calculating each characteristic, the more prepared they will be to compare boxplots and answer critical questions about the data represented in a boxplot.

## Stage 2 - Evidence

Learning Goals  
Measured:

\*HSS.ID.A.4

**Success Criteria** (e.g.. Learning progression, rubric, proficiency scale, etc.)

| Skill:                                   | Level 3:                                                                                     | Level 2:                                                                          | Level 1:                                                                        |
|------------------------------------------|----------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|---------------------------------------------------------------------------------|
| Calculating characteristics of a boxplot | Accurately calculate median, IQR, boundaries and quartiles                                   | Accurately calculate some but not all markers on a boxplot                        | Calculate only median and quartiles                                             |
| Make and compare boxplots                | Accurately draw and label 2 boxplots on the same grid. Identify similarities and differences | Identify similarities and differences of 2 boxplots that have already been drawn. | Draw two boxplots on the same grid but not able to answer similarity questions. |

**Sample Assessment** (e.g. Performance tasks, anchor of student work, common assessment etc.)

**SWEAT: Measure standard deviation, identify outliers, and construct a boxplot using the five-number summary**

### Unit Assessment

1. In a September 28, 2008, article titled "Letting Our Fingers Do the Talking," the New York Times reported that Americans now send more text messages than they make phone calls. According to a study by Nielsen Mobile, "Teenagers ages 13 to 17 are by far the most prolific texters, sending or receiving 1,742 messages a month." Mr. Williams, a high school statistics teacher, was skeptical about the claims in the article. So he collected data from his first-period statistics class on the number of text messages and calls they had sent or received in the past 24 hours. Here are the texting data:

---

|   |     |    |    |    |    |    |   |    |    |   |    |    |
|---|-----|----|----|----|----|----|---|----|----|---|----|----|
| 0 | 7   | 1  | 29 | 25 | 8  | 5  | 1 | 25 | 98 | 9 | 0  | 26 |
| 8 | 118 | 72 | 0  | 92 | 52 | 14 | 3 | 3  | 44 | 5 | 42 |    |

---

(a) Make a boxplot of these data by hand. Be sure to check and identify any outliers.

(b) Do these data support the claim in the article about the number of texts sent by teens? Justify your answer with appropriate evidence.

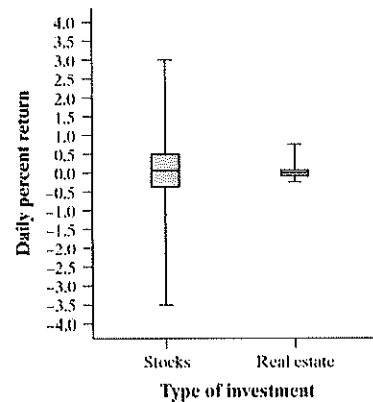
2. Here are the scores of Mrs. Liao's students on their first statistics test:

|    |    |      |    |      |      |      |    |      |      |    |
|----|----|------|----|------|------|------|----|------|------|----|
| 93 | 93 | 87.5 | 91 | 94.5 | 72   | 96   | 95 | 93.5 | 93.5 | 73 |
| 82 | 45 | 88   | 80 | 86   | 85.5 | 87.5 | 81 | 78   | 86   | 89 |
| 92 | 91 | 98   | 85 | 82.5 | 88   | 94.5 | 43 |      |      |    |

(a) Make a boxplot of the test score data by hand. Be sure to check and identify any outliers.

(b) How did the students do on Mrs. Liao's first test? Justify your answer.

3. Should you put your money into a fund that buys stocks or a fund that invests in real estate? The boxplots compare the daily returns (in percent) on a "total stock market" fund and a real estate fund over a year ending in November 2007.



(a) Read the graph: about what were the highest and lowest daily returns on the stock fund?

(b) Read the graph: the median return was about the same on both investments. About what was the median return?

(c) What is the most important difference between the two distributions?

|                                                 |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
|-------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                                                 |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| <b>Stage 3 – Learning Plan</b>                  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| Learning Goals Addressed:<br><u>*HSS.ID.A.4</u> | <b>Sample Assignment:</b> Students are provided a set of data. They are asked to calculate the standard deviation and generate a boxplot that best represents the data. Students can either do their calculations by looking at formulas or by having the formulas memorized. Once the calculations are completed, they would need to accurately place the results on a boxplot. They need to take care to put the boxplot on a grid so all the values can be read accurately. After finishing this process, students will answer a series of questions related to the data. The answers can all be observed from the boxplot they just completed. |
|                                                 | <b>Differentiated Approaches:</b><br>At the highest level of understanding, a student will be able to complete a boxplot if given a set of data. That can be differentiated by providing different levels of information along the way. The teacher can provide step by step support or they can just provide the assistance for the parts that have been forgotten. An example can be given as reference and or students can work in heterogeneous groups to help each other remember the many components of the process.                                                                                                                         |

|                                                |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
|------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Stage 1 Desired Results</b>                 |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| <b>Unit Title: Trend Lines and Correlation</b> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| ESTABLISHED LEARNING GOALS<br><u>*8.SPA.2</u>  | <i>Transfer</i>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
|                                                | <i>Students will be able to independently use their learning to...</i> <ul style="list-style-type: none"> <li>Describe scatter plots by direction, form, strength and visible outliers</li> <li>Estimate and calculate the correlation '<math>r</math>' between two quantitative variables</li> <li>Interpret '<math>r</math>' and discuss association vs. causation</li> <li>Use trend lines to model and predict outcomes.</li> <li>Calculate residuals from the trendline equation</li> </ul> |
|                                                | <i>Meaning Making</i>                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |

|  |                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                           |
|--|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|  | <b>UNDERSTANDINGS</b><br><i>Students will understand that...</i><br>*Sets of points can be categorized based on appearance.<br>*Equations can be created to represent data points.<br>*Data sets can be compared using equations and predictions. | <b>ESSENTIAL QUESTIONS</b><br>*Can the data be described as increasing, decreasing or scattered?<br>*How close is your trend line to the data?<br>*What future values can you predict?                                    |
|  | <b>Acquisition</b>                                                                                                                                                                                                                                |                                                                                                                                                                                                                           |
|  | <i>Students will know ...</i><br>*How to categorize data sets<br>*How to predict future values<br>*How accurate their predictions are                                                                                                             | <i>Students will be skilled at...</i><br>*Comparing data sets to lines of fit<br>*Creating lines of fit<br>*Calculating 'r' the correlation between data points and a line of fit<br>*Make predictions from a line of fit |

5-10 sentence summary of key learning in this unit.

Students will start by observing sets of data plotted as points. They will look for characteristics like linear vs. non-linear, positive correlation vs. negative correlation and no correlation. Once students determine that a data set has a linear representation, they will go through the process of finding the line and linear equation that best represent the data. Students will use correlation calculations to argue the accuracy of their line and then predict future values using the linear equation they created.

## Stage 2 - Evidence

| Learning Goals Measured:<br><u>*8.SPA.2</u> | Success Criteria (e.g.. Learning progression, rubric, proficiency scale, etc.) |                                                                                                                                                                                               |                                                                                                                                         |                                                                                           |
|---------------------------------------------|--------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------|
|                                             | Skill:                                                                         | Level 3:                                                                                                                                                                                      | Level 2:                                                                                                                                | Level 1:                                                                                  |
|                                             | Categorizing sets of data                                                      | Students accurately distinguish between scattered, linear, non-linear, increasing and decreasing sets                                                                                         | Students can distinguish between linear and non-linear but not increasing vs. decreasing.                                               | Students treat all points as linear and use random lines through the points.              |
|                                             | Finding and writing equations for line of best fit                             | Students visually place their line of fit where the number of points above and below are about the same with as many on the line as possible. Equation is written from two points on the line | Placement of line is arguably accurate but does not fit the half above half below rule. Equation is written but not exact to line drawn | Line is drawn randomly and equation does not match the line drawn.                        |
|                                             | Using residual 'r' to defend a line of fit                                     | Students accurately calculate the residual between the data points and their line of                                                                                                          | Student can find the residuals but is not fluent at calculating the correlation                                                         | Student understands the difference between the data points and the line of fit but is not |

|                                                 |                                                                 |                                                             |                                                   |
|-------------------------------------------------|-----------------------------------------------------------------|-------------------------------------------------------------|---------------------------------------------------|
|                                                 | fit and then use that information to calculate the correlation. |                                                             | able to calculate the residual or the correlation |
| Using equation of fit to predict a future value | Students accurately predict a future value from their equation. | Student accurately approximates a future value from a graph | Students can only approximate a future value      |

**Sample Assessment** (e.g. Performance tasks, anchor of student work, common assessment etc.)

Quiz #2--Best Fit + Error

Statistics

version 1

Name \_\_\_\_\_

Date \_\_\_\_\_

- I. Find the median median line of best fit from the data below.  
 II. Calculate the margin of error using the Root Mean Square formula.

'x' is the number of hours spent studying each week.

'y' is the number of hours spent watching TV each week.

| x       | y  | predicted<br>y | residual | residual<br>squared |
|---------|----|----------------|----------|---------------------|
| 10      | 15 |                |          |                     |
| 12      | 14 |                |          |                     |
| 14      | 13 |                |          |                     |
| 20      | 12 |                |          |                     |
| 25      | 11 |                |          |                     |
| 28      | 10 |                |          |                     |
| 30      | 9  |                |          |                     |
| 32      | 8  |                |          |                     |
| Sum =   |    |                |          |                     |
| (n - 2) |    |                |          |                     |
| Error = |    |                |          |                     |

$$m = \boxed{\phantom{000000}}$$

$$b1 = \underline{\hspace{2cm}}$$

$$b2 = \underline{\hspace{2cm}}$$

$$b3 = \underline{\hspace{2cm}}$$

$$b = \boxed{\phantom{000000}}$$

$$y = \boxed{\phantom{000000}}$$



|                                                         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
|---------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Learning Goals Addressed:</p> <p><u>*8.SPA.2</u></p> | <p><b>Sample Assignment:</b></p> <p>Students are either given or are asked to collect data that can be plotted on a coordinate plane. They are then asked to use a method of choice to create a line of fit. (median line of fit, mean line of fit, recursion tool) Once the line is drawn on the same grid as the data points, students move on to calculating the residual between their line and every data point. Once each residual is found, students calculate the correlation or the error. Students finish the lesson by answering a number of questions about predictability or interpreting their line of fit.</p>                                                                                               |
|                                                         | <p><b>Differentiated Approaches:</b></p> <p>Some students are well versed in writing the equation of a line given two points but many are not so those students that need assistance can be given either an example or can be retaught how to write the equation of a line from two points. Using a spreadsheet may also be helpful for students to keep track of their calculations as some students can program the cells in the spreadsheet to make the calculations whereas other students may need a spreadsheet provided that already have the cells programmed. As students progress, they will be able to do all of these tasks themselves or at least be able to describe what the spreadsheet is calculating.</p> |

| Stage 1 Desired Results                                     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|-------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Unit Title: Standard Normal Distribution</b>             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| <p>ESTABLISHED LEARNING GOALS</p> <p><u>*HSS.ID.A.4</u></p> | <i>Transfer</i>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|                                                             | <p><i>Students will be able to independently use their learning to...</i></p> <ul style="list-style-type: none"> <li>• Label the normal curve for both z and x values using the 68-95-99.7 rule</li> <li>• Use the Empirical rule to estimate Normal probabilities</li> <li>• Calculate and interpret a z-score</li> <li>• Generate a distribution curve from data</li> <li>• Find boundary points given probabilities</li> <li>• Calculate a confidence interval for a Population Proportion and Population Mean</li> </ul> |
|                                                             | <i>Meaning Making</i>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |

|  |                                                                                                                                                                                                                                                                    |                                                                                                                                                                                                                      |
|--|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|  | <b>UNDERSTANDINGS</b><br><i>Students will understand that...</i><br>*Some data can and some can not be represented with a normal distribution<br>*Normal distribution follows the empirical rule<br>*The area the normal curve can also be viewed as a probability | <b>ESSENTIAL QUESTIONS</b><br>*What are the primary components in a normal distribution curve?<br>*How can you use a normal curve to analyze data?<br>*How are probabilities related to a normal distribution curve? |
|  | <b>Acquisition</b>                                                                                                                                                                                                                                                 |                                                                                                                                                                                                                      |
|  | <i>Students will know...</i><br>*How to use a distribution curve to represent data.<br>*How to read a distribution curve as a probability tool.<br>*How to compare a data set to that of a normal distribution curve.                                              | <i>Students will be skilled at...</i><br>*Calculating standard deviation<br>*Calculating z-scores<br>*Calculating probabilities<br>*Analyzing data as it compares to a normal curve.                                 |

5-10 sentence summary of key learning in this unit.

Students will start by learning the empirical rule and how it is the percentage of data under a normal curve. They will learn how to use standard deviation to break up the data and how to compare their results with that of a normal curve. They will learn the relationship between a histogram and the normal curve as they calculate the percentage of data within each block. Finally, students will learn how to use a normal distribution of data to answer questions about probability, area and normalcy.

## Stage 2 - Evidence

| Learning Goals Measured:<br>* <u>HSS.ID.A.4</u> | Success Criteria (e.g.. Learning progression, rubric, proficiency scale, etc.) |                                                                                                                                                                                                    |                                                                                                                                                                                           |                                                                                             |
|-------------------------------------------------|--------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------|
|                                                 | Skill:                                                                         | Level 3:                                                                                                                                                                                           | Level 2:                                                                                                                                                                                  | Level 1:                                                                                    |
|                                                 | Reproduce a normal distribution curve                                          | Students reproduce a normal distribution curve accurately labeling the mean and all 6 deviations from the mean. All percentages are labeled in each region and the curve appears to be symmetrical | Student is able to reproduce a normal curve with mean and 3 deviations above and below the mean but does not label the percentages for each region. Curve may or may not look symmetrical | Student is able to reproduce a normal curve with the mean but no deviations or percentages. |
|                                                 | Create a normal distribution of data using mean and standard deviation.        | Student calculates the mean and standard deviation of the data. Student creates a distribution curve showing the calculated mean and 3 standard                                                    | Student calculates the mean and standard deviation of the data. Student creates a distribution curve showing the calculated mean and 3 standard                                           | Student calculates the mean and accurately labels it on a distribution curve.               |

|  |                                                                                              |                                                                                                 |  |
|--|----------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------|--|
|  | deviations above and below the mean. Percentages for each region are calculated and labeled. | deviations above and below the mean but percentages for each region are not accurately labeled. |  |
|--|----------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------|--|

**Sample Assessment** (e.g. Performance tasks, anchor of student work, common assessment etc.)

Name \_\_\_\_\_ Per: \_\_\_\_\_ Date \_\_\_\_\_

### Normal Distributions Test

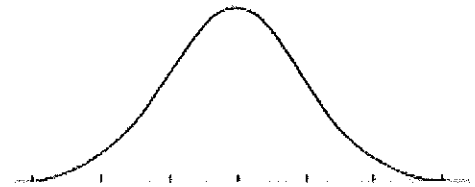
A set of data with a mean of 45 and a standard deviation of 8.3 is normally distributed. Find each value, given its distance from the mean.

1. +1 standard deviation from the mean
2. +3 standard deviations from the mean
3. -1 standard deviation from the mean
4. -2 standard deviations from the mean

Sketch a normal curve for each distribution. Label the  $x$ -axis at one, two, and three standard deviations from the mean.

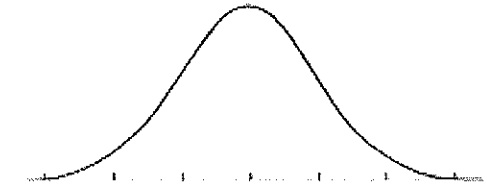
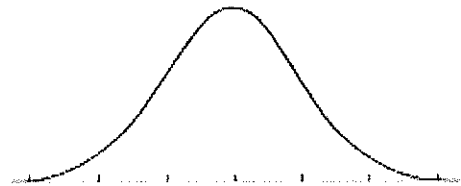
5. mean = 95; standard deviation = 12

6. mean = 100; standard deviation = 15



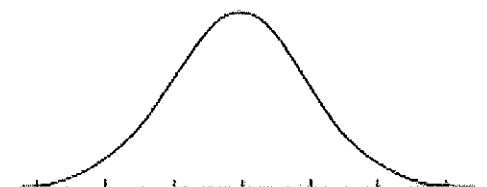
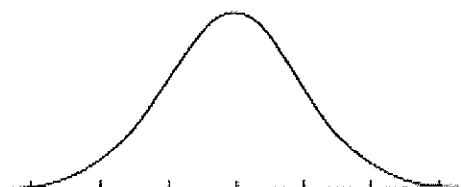
7. mean = 60; standard deviation = 6

8. mean = 23.8; standard deviation = 5.2



9. mean = 676; standard deviation = 60

10. mean = 54.2; standard deviation = 12.3



A set of data has a normal distribution with a mean of 5.1 and a standard deviation of 0.9. Find the percent of data within each interval.

11. Sketch a normal curve for the distribution.

12. between 6.0 and 6.9

13. greater than 6.9

14. between 4.2 and 6.0

15. less than 4.2

16. less than 5.1

17. between 4.2 and 5.1



18. Test scores are normally distributed with a mean of 76 and a standard deviation of 10.

a. In a group of 230 tests, how many students score above 96?

b. In a group of 230 tests, how many students score below 66?

c. In a group of 230 tests, how many students score within one standard deviation of the mean?

19. The number of nails of a given length is normally distributed with a mean length of 5.00 in. and a standard deviation of 0.03 in.

a. Find the number of nails in a bag of 120 that are less than 4.94 in. long.

b. Find the number of nails in a bag of 120 that are between 4.97 and 5.03 in. long.

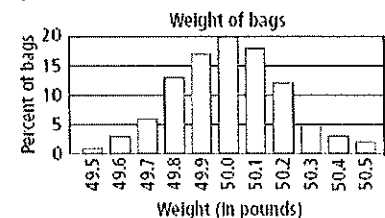
c. Find the number of nails in a bag of 120 that are over 5.03 in. long.

20. The actual weights of bags of pet food are normally distributed. The mean of the weights is 50.0 lb, with a standard deviation of 0.2 lb. Use the graph for a - c.

a. About what percent of bags of pet food weigh less than 49.8 lb?

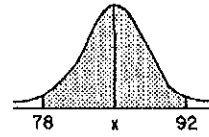
b. In a group of 250 bags, how many would you expect to weigh more than 50.4 lb?

c. In a group of 50 bags, how many would you expect to be within 1.5 standard deviations of the mean?



21. In the accompanying diagram, the shaded area represents approximately 95% of the scores on a standardized test. If these scores ranged from 78 to 92,

- What is the mean?
- What is the standard deviation?



22. A machine is used to put bolts into boxes. It does so such that the actual number of bolts in a box is normally distributed with a mean of 106 and a standard deviation of 2.

- Draw and label the Normal curve from the information.
- What percentage of boxes contain more than 104 bolts?
- What percentage of boxes contain more than 110 bolts?
- What percentage of boxes contain less than 108 bolts?
- What percentage of boxes contain less than 100 bolts?
- What percentage of boxes contain between 102 and 112 bolts?
- What percentage of boxes contain between 100 and 106 bolts?



23. The heights of the people of the planet Ixx are normally distributed with a mean of 40 inches and a standard deviation of 5 inches. [They are a vertically diverse people.]

- Draw and label the Normal curve from the information.
- 97.5% of Ixxians are over \_\_\_\_\_ inches tall? 97.5%
- 16% of Ixxians are over \_\_\_\_\_ inches tall?
- 50% of Ixxians are under \_\_\_\_\_ inches tall?
- 97.5% of Ixxians are under \_\_\_\_\_ inches tall?
- the most "average" 68% of Ixxians are between \_\_\_\_\_ and \_\_\_\_\_ inches tall?
- 84% of Ixxians are over \_\_\_\_\_ inches tall?



|                                                 |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
|-------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Learning Goals Addressed:<br><u>*HSS.ID.A.4</u> | <b>Sample Assignment:</b><br>Students are given a normal distribution curve completely labeled for reference and a set of data. They work in small groups to calculate the mean and standard deviation of the data. They then generate their own distribution curve using the mean and standard deviation they just calculated. Students count the frequency in each region and label the curve accordingly. After the frequency is distributed, each student calculates the percentage within each region. Using all this information, students answer a number of questions related to the data set.                                                                  |
|                                                 | <b>Differentiated Approaches:</b><br>This unit is mostly visual representation of data so there are many entry points for students. Some students may have the normal distribution memorized and some may need the normal curve provided for them. Most students are somewhere in between so the normal distribution critical features can be shared on an as needed basis. Some students can be provided with the empirical rule while other know that part and need to know the formula for calculating standard deviation. Some students need help identifying the frequencies and other only need help calculating the percentages after the frequencies are found. |

| Stage 1 Desired Results                          |                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                                                                                                                                                                                                                          |
|--------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Unit Title: Confidence Interval                  |                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                                                                                                                                                                                                                          |
| ESTABLISHED LEARNING GOALS<br><u>*HSS-IC.B.4</u> | <i>Transfer</i>                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                                                                                                                                                                                                                          |
|                                                  | <i>Students will be able to independently use their learning to...</i> <ul style="list-style-type: none"> <li>· Calculate a confidence interval for a Population Proportion and Population Mean</li> <li>· Compare data to a Normal distribution using mean and standard deviation.</li> <li>· Use the Empirical rule to estimate Normal probabilities</li> <li>· Calculate and interpret a z-score</li> <li>· Produce a Normal Probability Plot</li> </ul> |                                                                                                                                                                                                                                                                                          |
|                                                  | <i>Meaning Making</i>                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                                                                                                                                                                                                          |
|                                                  | <b>UNDERSTANDINGS</b><br><i>Students will understand that...</i> <ul style="list-style-type: none"> <li>*Sample size impacts the accuracy of a prediction</li> <li>*Sample means can be used to predict population means</li> <li>*z-scores represent the number of standard deviations from the mean</li> </ul>                                                                                                                                            | <b>ESSENTIAL QUESTIONS</b> <ul style="list-style-type: none"> <li>*What can be done to increase the level of confidence?</li> <li>*What z*-score corresponds to 90-95-99% confidence?</li> <li>*What is the difference between a population mean and a population proportion?</li> </ul> |

|  |                                                                                                                                                                                                                                 |                                                                                                                                                                                                 |
|--|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|  |                                                                                                                                                                                                                                 |                                                                                                                                                                                                 |
|  | <b>Acquisition</b>                                                                                                                                                                                                              |                                                                                                                                                                                                 |
|  | <i>Students will know...</i><br><i>*How to find a range within a certain level of confidence.</i><br><i>*Predict a population mean from a sample mean.</i><br><i>*Predict a population proportion from a sample proportion.</i> | <i>Students will be skilled at...</i><br><i>*Calculating a confidence interval</i><br><i>*Using formulas to calculate z*</i><br><i>*Using a normal distribution to determine probabilities.</i> |

5-10 sentence summary of key learning in this unit.

Students will learn the significance of a population mean vs. a sample mean. They will learn how to take an attainable data point like a sample mean and use it to predict what the population mean is. More importantly, they will learn how to represent their answer within a desired level of confidence. They will get to the point where they can increase the sample size in order to increase their level of confidence. Students will learn that their answer will be presented as a range instead of an exact value so that it falls within their calculated level of confidence.

## Stage 2 - Evidence

Learning Goals

Measured:

\*HSS-IC.B.4

**Success Criteria** (e.g.. Learning progression, rubric, proficiency scale, etc.)

| Skill:                          | Level 3:                                                                                                | Level 2:                                             | Level 1:                                                        |
|---------------------------------|---------------------------------------------------------------------------------------------------------|------------------------------------------------------|-----------------------------------------------------------------|
| Calculate a confidence interval | Students know and use formulas accurately create an interval that meets the desired level of confidence | Student use formulas to create a confidence interval | Students show an interval but may have made calculation errors. |

**Sample Assessment** (e.g. Performance tasks, anchor of student work, common assessment etc.)

Test—Confidence Interval  
Statistics—Spring

Name \_\_\_\_\_

|                                                                                                                                                                                                                                                                                         |                                                                                                        |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------|
| 1) A questionnaire of study time was given to a random sample of high school seniors. Each senior was asked to record and report the amount of time they spent studying each night. The sample of 180 seniors resulted in an average of 3.5 hours with standard deviation of .95 hours. |                                                                                                        |
| a) Calculate the 90% confidence interval for the mean amount of time seniors spend studying nightly.                                                                                                                                                                                    | b) What is the margin of error for the 90% confidence interval?                                        |
| c) If 250 seniors were surveyed, what would the margin of error be with a 90% level of confidence?                                                                                                                                                                                      | d) How many seniors should you survey if you want a 5% margin of error with a 90% level of confidence? |

|                                                                                                                                                                                                                           |                                                                   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 2) A sample of student test scores yields the following:                                                                                                                                                                  |                                                                   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| <table border="1"> <tr> <td>88</td> <td>85</td> <td>92</td> <td>81</td> <td>69</td> <td>71</td> <td>75</td> </tr> <tr> <td>77</td> <td>92</td> <td>88</td> <td>75</td> <td>66</td> <td>72</td> <td>83</td> </tr> </table> |                                                                   | 88 | 85 | 92 | 81 | 69 | 71 | 75 | 77 | 92 | 88 | 75 | 66 | 72 | 83 |
| 88                                                                                                                                                                                                                        | 85                                                                | 92 | 81 | 69 | 71 | 75 |    |    |    |    |    |    |    |    |    |
| 77                                                                                                                                                                                                                        | 92                                                                | 88 | 75 | 66 | 72 | 83 |    |    |    |    |    |    |    |    |    |
| a) Calculate the sample mean.                                                                                                                                                                                             | b) Calculate the sample standard deviation.                       |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| c) Calculate the 90% confidence interval for the population mean.                                                                                                                                                         | d) Calculate the 95% confidence interval for the population mean. |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| e) Calculate the 99% confidence interval for the population mean.                                                                                                                                                         | f) Identify the margin of error for all 3 confidence levels.      |    |    |    |    |    |    |    |    |    |    |    |    |    |    |

|                                                                                 |                                                                                  |
|---------------------------------------------------------------------------------|----------------------------------------------------------------------------------|
| 3) Indicate whether each statement is true or false and give a reason for each. |                                                                                  |
| a) As the confidence interval increases, the margin of error decreases.         | b) As the sample size increases, the margin of error increases.                  |
| c) As the standard deviation increases, the margin of error increases.          | d) If you want to decrease the margin of error, what could you do in your study? |



4) An insurance company checks police records on 582 accidents selected at random and notes that teenagers were at the wheel in 91 of them.

a) Calculate the 90% confidence interval for the proportion of teenagers involved in accidents.

b) What is the margin of error for the 90% confidence interval you just predicted?

c) If 1000 police records were checked and the proportion stayed the same, what would the margin of error be with a 90% level of confidence?

d) How many police records would need to be checked if you want a 5% margin of error with a 90% level of confidence?

5) In a random survey of 226 college students, 20 reported being "only" children (with no siblings).

a) Construct a 95% confidence interval for the proportion of students nationwide who are only children.

b) Explain what your interval means.

c) Explain what would change if you were to calculate a 99% confidence interval for the proportion of students nationwide who are only children.

d) What impact does the number of students surveyed have on your proportion prediction?

6) A random sample of 168 students were asked how many songs were in their digital music library and what fraction of them were legally purchased. Overall, they reported having a total of 117,079 songs, of which 23.1% were legal.

a) Construct a 99% confidence interval for the fraction of legal digital music.

b) Calculate a good estimate of legally attained songs in a student's digital music library.

c) What is your margin of error?

d) What would your margin of error be if you asked 1500 random students?

|                                                         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
|---------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Learning Goals Addressed:<br/>*<u>HSS-IC.B.4</u></p> | <p><b>Sample Assignment:</b></p> <p>In small groups, students conduct a survey in class that can be used to predict something for their school. Students use the sampling to calculate a confidence interval so that when they present their findings, they can do so with a certain level of confidence. Students finish up by answering a series of questions about their results and present their findings to the rest of the class via presentation or poster.</p>                                                                                                                                                                                                                                                                                                                                                              |
|                                                         | <p><b>Differentiated Approaches:</b></p> <p>Not only does this unit require formulas, but each component in the formula is unique. Because of this, students may have different entry points. At the highest entry point, students will know the formulas, what every component is and how to use them. For other students, there will be a variety of entry points. Some students may need to be given the formulas while other students may need to be given the formulas and assistance remembering what each component represents. Still others may need help using the formulas to make their calculations. These different entry points can be largely addressed while students are in their small group however, if there are individual needs that the group is not providing, teachers will need to provide assistance.</p> |

### Instructional Materials:

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Suggested textbook(s), materials, equipment and resources</p> <ul style="list-style-type: none"> <li>● Larson and Hostetler, <u>Trigonometry</u>: Houghton Mifflin; 5<sup>th</sup> edition, 2001</li> <li>Starnes and Tabor, <u>Statistics and Probability with Applications</u>: Bedford, Freeman &amp; Worth; 3<sup>rd</sup> edition, 2017</li> <li>● Books and materials have been previously approved by the board</li> <li>● Graphing Calculators, Computer software (e.g., Desmos, Google sheets), Online research</li> </ul> |
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